Research Statement

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My research interests, in the broadest sense, are in the algebraic and geometric structures underlying physical theories. My previous work has focused mainly on applications of Lie (super)algebra cohomology in mathematical physics, in particular in supergravity and symplectic mechanics. In my PhD thesis, *Spencer cohomology, supersymmetry and the structure of Killing superalgebras* [1] and related publications [2–5], I expanded upon a body of work by my supervisor José Figueroa O'Farrill (JMF) and others studying the supersymmetry algebras of supersymmetric supergravity backgrounds via Spencer cohomology [6–11], an approach which not only provides insights into the structure of these superalgebras but also gives one a geometrical notion of supersymmetry which goes beyond supergravity, the consequences of which we have only begun to explore. In my work in symplectic mechanics [12, 13], I proved an often stated but never fully justified version of a classic theorem on homogeneous symplectic spaces, and an extension of this work is still ongoing.

In this statement, I will briefly provide some background for and an overview of my research output to date and then describe some research projects which I am interested in pursuing, encompassing problems in geometry, supersymmetry and gravitational theory across various regimes.

Spencer cohomology and supersymmetry

Background theory A supersymmetric background in a supergravity theory is a solution in which all fermions vanish and which is preserved by at least one of the supersymmetries of the theory, meaning in particular that the supersymmetry variation of the gravitino with respect to some local spinor parameter ϵ vanishes. Mathematically, this is interpreted as a first-order linear differential equation, the *Killing spinor equation* (KSE), which is generally of the form $\nabla\epsilon = \beta\epsilon$, where ϵ is understood to be a section of a spinor bundle, ∇ is a lift of the Levi-Civita connection and β is a 1-form with values in spinor endomorphisms (which is parametrised by the bosonic background fields). The solutions to this equation are known as *Killing spinors*.[1]

Killing spinors generate a Lie superalgebra known as a *Killing superalgebra* (KSA), which one interprets as a supersymmetry algebra of the background, and it has a particular structure: it is a *filtered subdeformation* of the Poincaré superalgebra[2] [1, 3, 8]. Such deformations are governed by a cohomology theory known as Spencer cohomology [14]; in the case at hand, elements of the Spencer cohomology group $\mathscr{H}^{2,2}$ of the Poincaré superalgebra play the role of "infinitesimal" filtered subdeformations which one may be able to "integrate" to full filtered deformations. While this perspective is somewhat abstract, an element of $\mathscr{H}^{2,2}$ can be interpreted more concretely as a spinor endomorphism-valued 1-form which one can identify with the β appearing in the KSE. A priori, this calculation only tells one about *maximally* supersymmetric backgrounds, but it can often be bootstrapped to provide much more information, especially about highly supersymmetric (>$\frac{1}{2}$BPS) backgrounds. In particular cases it actually completely recovers the form of the KSE, and it is sometimes even possible to recover the bosonic equations of motion by imposing integrability conditions [2, 7–10]. That is to say, using only the representation theory of the Poincaré superalgebra, one can, at least in certain cases, recover the bosonic sector of a supergravity theory without any prior knowledge of the field content, field equations or supersymmetry transformations of that theory.

My work In my first published work [2], which built upon my MSc thesis, we applied these methods to minimal $D = 5$ supergravity, determining the Spencer cohomology group $\mathscr{H}^{2,2}$ and finding that the KSE and bosonic equations of motion can be recovered, but we also found that the form of the KSE suggested by the calculation is more general than that of the supergravity theory, with additional terms proportional to a new bosonic field not present in the theory (also previously observed in $D = 6$ [10]). Nonetheless, the spinors satisfying this equation generate a KSA, and we showed that there are geometries supporting such spinors which are not supersymmetric supergravity backgrounds. Thus the geometric notion of "supersymmetry" – the admissibility of spinors generating a KSA – is more general than the one given by supergravity theory. One potential consequence of this is that the well-studied class of supersymmetric 5-dimensional black holes might also be enriched; see point 2 below.

[1] These are closely related to but not exactly the same as the Killing spinors studied in Riemannian geometry satisfying the equation $\nabla_X\epsilon = \lambda X \cdot \epsilon$ for all vector fields X and some $\lambda \in \mathbb{C}$, which we term *geometric* Killing spinors.

[2] That is, it is a filtered Lie superalgebra, and its associated graded Lie superalgebra is a ($\mathbb{Z}$-graded) subalgebra of Poincaré.

I extended this work in my PhD thesis, calculating $\mathscr{H}^{2,2}$ for arbitrary $\mathcal{N}$-extended Poincaré superalgebras in $D = 5$ and $D = 6$ and including extension by the R-symmetry algebra. In the minimal $D = 5$ case, I showed that when the R-symmetry is included, one discovers an even more general KSE, and by describing some families of maximally supersymmetric backgrounds, I demonstrated that the corresponding notion of supersymmetric geometry is much richer than that of supergravity. I also showed that $\mathscr{H}^{2,2} = 0$ in Type IIA; this is consistent with the non-existence of non-trivial *maximally* supersymmetric solutions in Type IIA supergravity, but more work is needed to understand what else this might have to tell us about the theory.

Parallel to these calculations, I developed a framework for understanding KSAs via Spencer cohomology in a general context – the full relationship had only previously been fully described in $D = 11$ supergravity [8]. I described the requirements on β for the solutions to the equation $\nabla\epsilon = \beta\epsilon$ on an arbitrary pseudo-Riemannian spin manifold to define a KSA, as well what Spencer cohomology tells one about this and the notion of supersymmetric geometry it defines. The theoretical results of this effort and a set of 2-dimensional examples comprise my two most recent publications [3, 4]. In a recent preprint [5], also developing work from my thesis, I went on to adapt this framework to *generalised spin* or *spin-G structures* [15, 16] (about which I also wrote the note [17]), showing that in this setting one naturally obtains KSAs which are "twisted" by R-symmetry, previously introduced as deformations of R-extended Poincaré superalgebras in work in $D = 6$ [10]. In my thesis, I argued that this gives the correct notion of KSA for gauged supergravity theories,[3] and I validated this using my explicit calculations in $D = 5$, which I intend to publish in the near future.

Future directions I detail below a number of potential research projects which follow on from the work described above, starting with those with the clearest path to publication.

1. **2-dimensional examples.** The examples described in [4] and other recent work [18] have proven dimension 2 to be an interesting proving ground for the Spencer cohomology approach to supersymmetry. Having done some preliminary work on examples with $\mathcal{N}$-extended supersymmetry and R-symmetry, I intend to write a paper in the near future describing some of their more interesting features.

2. **Supersymmetric black holes.** An ansatz of supersymmetry has been used for finding exotic black hole solutions, in particular in $D = 5$ [19]. It may be the case that new black hole solutions could be found using the more general notion of supersymmetry that we have developed. More ambitiously, one could attempt to classify *all* supersymmetric geometries under this definition, as was done for minimal $D = 5$ supergravity [20]. I have already carried out many relevant calculations, such as determining relations among the spinor bilinears, in my PhD thesis.

3. **Kaluza–Klein reduction.** Dimensional reduction of the Spencer data of the Poincaré superalgebra, i.e. obtaining $\mathscr{H}^{2,2}$ in a given number of spacetime dimensions from that in higher dimensions, remains poorly understood. Previous work on this problem stalled due to technical difficulties, but we now have many more explicit examples which could provide a fresh perspective on this problem. Moreover, a closer analysis of the 10-dimensional supergravity theories in the framework I have developed and a comparison with the 11-dimensional case could shed light on both this issue and the trivial result ($\mathscr{H}^{2,2} = 0$) in Type IIA.

4. **Geometric supergravity.** The supergravity formalism of Castellani–D'Auria–Fré [21] can be formulated in terms of (super-)Cartan geometry, which is closely linked with Spencer cohomology [22]. I would be interested in exploring connections between my previous work and this perspective, and especially in how the incorporation of higher gauge structures and L_∞ algebras [23, 24] might help to resolve some open questions about what Spencer cohomology can tell us about the gauge data of supergravity backgrounds. There are also connections with recent work on non-relativistic supergravity and strings [25, 26].

5. **Rigid supersymmetry.** Supersymmetric bosonic backgrounds of supergravity theories are known to support rigid supersymmetric field theories obtained by "freezing out" gravitational degrees of freedom (via Festuccia–Seiberg [27]). The existence of more general supersymmetric backgrounds with KSAs opens up the possibility that there may be novel rigid supersymmetric field theories which do not arise this way. It may be possible to write down such new theories by studying the representation theory of the KSAs.

[3] That is, theories in which the R-symmetry group is gauged, so that the gravitini carry colour charge. To my knowledge, their KSAS have not previously been described.

Homogeneous symplectic geometry

Background theory In the symplectic formalism of Hamiltonian dynamics, a classic result of Kostant–Kirillov–Souriau (KKS) [28–30] says that the homogeneous symplectic spaces of a compact connected Lie group G, which Souriau considers to be a classical counterpart to elementary particles, are locally isomorphic to coadjoint orbits of a one-dimensional central extension $\widehat{G}$ of G. The classic references for this theorem all assume (either implicitly or explicitly) that G is simply connected, but this result is often stated without this assumption.

My work After discovering that there was no published proof of the KKS result without this assumption, [4] motivated by an application to non-relativistic dynamics [32], JMF and I identified that the difficulty when G is not simply connected is in showing that one can construct $\widehat{G}$ from homological data associated to the homogeneous space. In our preprint [12], we were able to resolve this issue using some existing results on the existence of central extensions of Lie groups [33, 34]. I also presented a summary of this work a conference paper [13].

Future directions We have more recently considered how this result might be generalised to the case where the group action only preserves the symplectic form up to rescaling, and this work has touched on the areas of Poisson, contact and Jacobi geometry. We hope to complete this work in the near future. I would also be interested in continuing to work on related problems in symplectic geometry and dynamics (both relativistic and non-relativistic).

Futher interests

I am interested in exploring topics across mathematical physics at the intersection of supersymmetry, gravity, homological algebra and geometry. In recent years, a number of different workshops, seminars and reading groups have drawn my interest towards, among other topics, non-perturbative aspects of supersymmetric quantum field theories, mathematical general relativity, generalised geometry and non-relativistic physics (in particular Newton–Cartan and Carrolian (super)gravity), and I would also be interested in working in these areas.

References

[1] A. Beckett. *Spencer cohomology, supersymmetry and the structure of Killing superalgebras*. PhD Thesis, U. of Edinburgh. 2024.

[2] A. Beckett and J. Figueroa-O'Farrill. "Killing superalgebras for lorentzian five-manifolds". In: *JHEP* 07 (2021), p. 209.

[3] A. D. K. Beckett. "Killing (super)algebras associated to connections on spinors". In: *SIGMA Symmetry Integrability Geom. Methods Appl.* 21 (2025), Paper No. 081.

[4] A. D. K. Beckett. "Killing superalgebras in 2 dimensions". In: *SIGMA Symmetry Integrability Geom. Methods Appl.* 21 (2025), Paper No. 082.

[5] A. D. K. Beckett. *Killing (super)algebras for generalised spin manifolds*. 2025. arXiv: `2511.07246 [math.DG]`.

[6] J. Figueroa-O'Farrill and A. Santi. "Eleven-dimensional supergravity from filtered subdeformations of the Poincaré superalgebra". In: *J. Phys. A* 49.29 (2016), p. 295204.

[7] J. Figueroa-O'Farrill and A. Santi. "Spencer cohomology and 11-dimensional supergravity". In: *Commun. Math. Phys.* 349 (2017), pp. 627–660.

[8] J. Figueroa-O'Farrill and A. Santi. "On the algebraic structure of Killing superalgebras". In: *Adv. Theor. Math. Phys.* 21.5 (2017), pp. 1115–1160.

[9] P. de Medeiros et al. "Killing superalgebras for Lorentzian four-manifolds". In: *JHEP* 06 (2016), p. 106.

[10] P. de Medeiros et al. "Killing superalgebras for lorentzian six-manifolds". In: *J. Geom. Phys.* 132 (2018), pp. 13–44.

[11] A. Santi. "Remarks on highly supersymmetric backgrounds of 11-dimensional supergravity". In: *Geometry, Lie theory and applications—the Abel Symposium 2019*. Vol. 16. Abel Symp. Springer, Cham, 2022, pp. 253–277. ISBN: 978-3-030-81295-9.

[12] A. Beckett and J. Figueroa-O'Farrill. *Symplectic Actions and Central Extensions*. 2022. arXiv: `2203.07405 [math.SG]`.

[13] A. Beckett. "Homogeneous Symplectic Spaces and Central Extensions". In: *41st International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering*. July 2022. arXiv: `2207.03231 [math.SG]`.

[14] S.-J. Cheng and V. G. Kac. "Generalized Spencer cohomology and filtered deformations of $\mathbf{Z}$-graded Lie superalgebras". In: *Adv. Theor. Math. Phys.* 2.5 (1998). [Erratum: Adv.Theor.Math.Phys.8.5, 697–709 (2004)], pp. 1141–1182.

[15] S. J. Avis and C. J. Isham. "Generalized Spin Structures on Four Dimensional Space-Times". In: *Commun. Math. Phys.* 72 (1980), p. 103.

[16] B. Janssens. "Generalised Spin Structures in General Relativity". In: *Annales Henri Poincare* 19.5 (2018), pp. 1587–1610.

[17] A. D. K. Beckett. *Notes on generalised spin structures*. 2025. arXiv: `2511.03627 [math.DG]`.

[18] P. de Medeiros. *Poincaré superalgebras and triple systems*. 2024. arXiv: `2410.07950 [hep-th]`.

[19] J. P. Gauntlett et al. "Black holes of $D = 5$ supergravity". In: *Class. Quant. Grav.* 16 (1999), pp. 1–21.

[20] J. P. Gauntlett et al. "All supersymmetric solutions of minimal supergravity in five- dimensions". In: *Class. Quant. Grav.* 20 (2003), pp. 4587–4634.

[21] L. Castellani et al. *Supergravity and superstrings. A geometric perspective.* (3 Volumes). World Scientific Publishing Co., Inc., Teaneck, NJ, 1991, pp. xviii+xvi+xvi+2162. ISBN: 981-02-0673-9, 981-02-0674-7, 981-02-0675-5.

[22] A. Čap and K. Neusser. "On automorphism groups of some types of generic distributions". In: *Differ. Geom. Appl.* 27.6 (2009), pp. 769–779.

[23] K. Eder et al. *Poincaré Duality for Supermanifolds, Higher Cartan Structures and Geometric Supergravity*. 2025. arXiv: `2312.05224 [math-ph]`.

[24] D. Fiorenza et al. "Super-Lie n-algebra extensions, higher WZW models and super-p-branes with tensor multiplet fields". In: *Int. J. Geom. Methods Mod. Phys.* 12.2 (2015), pp. 1550018, 35.

[4] We were sometime later made aware of a different proof which appeared in a PhD thesis [31].

[25] R. Andringa et al. “3D Newton-Cartan supergravity”. In: *Classical Quantum Gravity* 30.20 (2013), pp. 205005, 19.
[26] E. A. Bergshoeff et al. “String theory and string Newton-Cartan geometry”. In: *J. Phys. A* 53.1 (2020), pp. 014001, 38.
[27] G. Festuccia and N. Seiberg. “Rigid Supersymmetric Theories in Curved Superspace”. In: *JHEP* 06 (2011), p. 114.
[28] J.-M. Souriau. *Structure des systèmes dynamiques*. Maîtrises de mathématiques. Dunod, Paris, 1970, pp. xxxii+414.
[29] B. Kostant. “Quantization and unitary representations. I. Prequantization”. In: *Lectures in Modern Analysis and Applications, III*. Vol. Vol. 170. Lecture Notes in Math. Springer, Berlin-New York, 1970, pp. 87–208.
[30] A. A. Kirillov. *Elements of the theory of representations*. Vol. Band 220. Grundlehren der Mathematischen Wissenschaften. Translated from the Russian by Edwin Hewitt. Springer-Verlag, Berlin-New York, 1976, pp. xi+315.
[31] Y. Voglaire. “Quantization of solvable symplectic symmetric spaces”. PhD thesis. Université Catholique de Louvain, Université de Reims Champagne-Ardenne, 2011.
[32] J. Figueroa-O’Farrill et al. “Lifshitz symmetry: Lie algebras, spacetimes and particles”. In: *SciPost Phys.* 14.3 (2023), Paper No. 035, 42.
[33] K.-H. Neeb. “A note on central extensions of Lie groups”. In: *J. Lie Theory* 6.2 (1996), pp. 207–213.
[34] G. M. Tuynman and W. A. J. J. Wiegerinck. “Central extensions and physics”. In: *J. Geom. Phys.* 4.2 (1987), pp. 207–258.